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Stating De Morgan’s First and Second Laws: Toward a Norm?

Jean-Paul K. Tsasa*

Abstract

This article documents the lack of consensus regarding the order of stating De Morgan’s laws in logic and mathematics textbooks. Based on a carefully selected collection of a score of textbooks, we show that 52% of authors state De Morgan’s laws using the conjunctive dominance approach, while the remaining 48% use the disjunctive dominance approach. This evidence remains robust when we expand our initial sample to a hundred textbooks. In this larger sample, we find that 53% of authors use the conjunctive dominance approach versus 47% of authors who use the disjunctive dominance. We propose a simple and tractable criterion that can alleviate the observed lack of consensus to date and foster the emergence of a standardized approach in the future.

Keywords: De Morgan’s laws, conjunction, disjunction, negation.

1 Introduction

This paper traces the modern origin of the initial formulation of De Morgan’s laws. In the literature, these laws are commonly attributed to the British mathematician Augustus De Morgan.¹ Over time, these laws have not only found a prominent place in the rich landscape of mathematical logic but have also proven their relevance across

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1. According to [Bocheński \(1961, p. 84\)](#) and [Kolmogorov and Yushkevich \(1992, p. 2\)](#), De Morgan’s laws have been known since the Middle Ages. However, as emphasized by [Johnstone \(1979, p. 479\)](#) and [Hobart and Richards \(2008, p. 283\)](#), De Morgan is credited with clearly stating these laws in the terms of symbolic logic.

a multitude of direct and indirect applications within diverse disciplines such as set theory, proof theory and constructive mathematics, probability calculus, artificial intelligence or logical aspects of theoretical computer science.²

In modern literature there is a pair of De Morgan’s laws. Surprisingly, despite the popularity of these laws, there seems to be a lack of a clear consensus about the order of stating them. For instance, some prominent authors, such as [Leung and Chen \(1967, p. 36\)](#), [Stolyar \(1971, p. 60\)](#), [Barwise \(1977, p. 25\)](#), [Quine \(1982, p. 57\)](#), [Epstein \(2011, p. 35\)](#), and, more recently, [Smith \(2022, p. 139\)](#) and [Hu \(2023, pp. 21\)](#), state De Morgan’s laws by starting with the negation of a conjunction; that is, by:

$$\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q},$$

then followed by the negation of a disjunction; that is, by: $\overline{P \vee Q} \equiv \overline{P} \wedge \overline{Q}$.

Simultaneously, other eminent authors, such as [Bocheński \(1960, pp. 20-21\)](#), [Enderton \(1977, p. 31\)](#), [Adamowicz and Zbierski \(1997, pp. 16-17\)](#), [Cori and Lascar \(2000, p. 31\)](#), [Chiswell and Hodges \(2007, p. 71\)](#), [Cunningham \(2016, p. 10\)](#), and [Allen and Hand \(2022, pp. 28-29\)](#), among others, state De Morgan’s laws by beginning with the negation of a disjunction; that is, by:

$$\overline{P \vee Q} \equiv \overline{P} \wedge \overline{Q},$$

then followed by the negation of a conjunction; that is, by: $\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q}$. Moreover, *The Concise Oxford Dictionary of Mathematics* (see [Clapham and Nicholson 2009, p. 118](#)), *The Edinburgh Dictionary of Philosophical Logic* ([Cook 2009, p. 86](#)), and *The Oxford Dictionary of Philosophy* ([Blackburn 2016, p. 126](#)), among others, also present De Morgan’s laws by beginning with the negation of a disjunction.

It is important to note that the purpose of this article is not to pass judgment on the best way to proceed concerning the order of stating De Morgan’s laws. Our humble goal is twofold. First, we seek to document the lack of consensus in the literature regarding the order of stating De Morgan’s laws in logic and mathematics textbooks. Indeed, it has been well-known since Descartes’s influential book,³ that a lack of consensus, even on basic things, can hinder harmony in scientific communication or academic discussions. On this subject, [Descartes \(1637, Part One\)](#) notes: “the diversity of our opinions does not come from the fact that some of us are

2. For further details, see, for instance, [Crowe and Hayes-Gill \(1998, ch. 1\)](#), [Holdsworth \(1987, ch. 2, 4\)](#), [Harris and Harris \(2016, ch. 2\)](#), and [da Silva et al. \(2020, p. 65, 70\)](#).

3. [Descartes’s \(1637\) A Discourse on the Method of Correctly Conducting One’s Reason and Seeking Truth in the Sciences](#) outlines fundamental principles in scientific reasoning.

more reasonable than others, but only from the fact that we have different ways of directing our thoughts, and do not take into account the same things.”

Second, we particularly aim to identify a simple criterion with a sufficiently robust archaeological foundation that can mitigate the observed lack of consensus in the future. Indeed, following [Foucault \(1969\)](#), the *archaeology* aims to analyze archives. Thus, to better engage with spoken or written things, Michel Foucault recommends developing a specific method, which he calls *archive*. By definition (see [Foucault 1969](#), pp. 177-178), the archive denotes the system that governs the appearance of statements as singular events. This prevents all things said or written from accumulating indefinitely in an amorphous multitude or following a linear progression without rupture.⁴

Thus, to derive a tractable criterion that can alleviate the observed lack of consensus to date and foster the emergence of a standardized approach in the future, this study refers to the initial modern formulation of De Morgan’s laws. We pay special attention to the order in which these laws were stated by the author to whom they are generally attributed; that is, Augustus De Morgan (see [De Morgan 1847](#)). Since there is a pair of De Morgan’s laws, this paper aims to explicitly specify which one was first stated in *Formal Logic*, historically constituting the *first law of De Morgan*, and which one was stated second, hence constituting the *second law of De Morgan*.

The remainder of the paper is organized as follows. Section 2 documents the lack of consensus about the order of statement of De Morgan’s laws in logic and mathematics textbooks. Section 3 briefly discusses the historical context in which De Morgan’s laws were introduced and provides a succinct review of his reference book; that is, *Formal Logic: The Calculus of Inference, Necessary and Probable*. Section 4 presents De Morgan’s laws. We explicitly identify the first and second laws, referring to the order in which they were stated by their author. Section 5 concludes.

2 Conjunctive Dominance versus Disjunctive Dominance

To ensure clarity in our exposition, we introduce two notions: *conjunctive dominance* and *disjunctive dominance*. Specifically, the conjunctive dominance approach

4. For more details, see [Foucault \(1969\)](#), ch. III, section 5). Also, see [Schumann \(2023\)](#) for a discussion on the archaeology of logic.

denotes the case where De Morgan’s laws are stated by beginning with the negation of a conjunction; that is:

$$\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q}.$$

Whereas, the disjunctive dominance approach refers to the case where De Morgan’s laws are stated by starting with the negation of a disjunction; that is:

$$\overline{P \vee Q} \equiv \overline{P} \wedge \overline{Q}.$$

This distinction helps to carefully organize the data presented in Table 1.

In Table 1, we provide empirical evidence of the lack of consensus regarding the order in which De Morgan’s laws are stated in logic and mathematics textbooks. To document this evidence, we carefully examined over a hundred textbooks,⁵ available in the libraries of McGill University, Sherbrooke University, the University of Montreal, and the University of Montreal in Quebec. As a baseline sample, we selected 25 textbooks. This baseline sample was characterized by its diversity, encompassing works known for their scientific rigor as well as those recognized for their pedagogical popularity.

The inclusion of technically advanced textbooks, known for their scientific rigor, enhances the credibility of our sample by presenting variations in interpretation within specialized academic literature. Simultaneously, the incorporation of popular pedagogical textbooks, often used in academic teaching, reflects the nuances and didactic choices that can influence the general understanding of these fundamental laws of logic. Consequently, such diversity captures different educational and scientific perspectives coexisting in the literary landscape in which De Morgan’s laws are typically used.

Furthermore, our baseline sample spans a sufficiently long period, from 1960 to 2022. This temporal scope provides a panoramic view of the evolving treatment of these laws over time, allowing us to grasp potential trends or changes in how these laws have been presented and taught.

5. See Appendix A for more details.

Table 1 – Order of stating De Morgan’s laws in a dataset of 25 logic and mathematics textbooks

Reference and target page	Publisher	Documentation 1	Documentation 2
		Statement 1: $\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q}$	Statement 1: $\overline{P \vee Q} \equiv \overline{P} \wedge \overline{Q}$
		Statement 2: $\overline{P \vee Q} \equiv \overline{P} \wedge \overline{Q}$	Statement 2: $\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q}$
		<i>Conjunctive dominance</i>	<i>Disjunctive dominance</i>
Bocheński (1960, pp. 20-21)	Springer Science		×
Hall (1966, p. 37)	Cambridge University Press		×
Leung and Chen (1967, p. 36)	Hong Kong University Press	×	
Stolyar (1971, p. 60)	The MIT Press	×	
Enderton (1972, p. 37)	Elsevier	×	
Barwise (1977, p. 25)	North Holland	×	
Enderton (1977, p. 31)	Academic Press		×
Quine (1982, p. 57)	Harvard University Press	×	
Hamilton (1988, p. 14)	Cambridge University Press	×	
Church (1996, p. 104)	Princeton University Press		×
Adamowicz and Zbierski (1997, pp. 16-17)	John Wiley & Sons		×
Halmos (1998, p. 20)	Springer Science		×
Hrbacek and Jech (1999, p. 14)	CRC Press	×	
Cori and Lascar (2000, p. 31)	Oxford University Press		×
Chiswell and Hodges (2007, p. 71)	Oxford University Press		×
Jech (2007, p. 78)	Springer Science		×
Epstein (2011, p. 35)	Princeton University Press	×	
Suppes (2012, p. 34)	Van Nostrand Reinhold	×	
Suppes and Hill (2012, pp. 98-99)	Dover Publications		×
Cunningham (2016, p. 10)	Cambridge University Press		×
Bernstein (2018, p. 22)	Princeton University Press	×	
Halmos (2018, p. 22)	Dover Publications	×	
Allen and Hand (2022, pp. 28-29)	The MIT Press		×
Smith (2022, p. 139)	Logic Matters, Cambridge	×	
Hu (2023, p. 21)	University of California Press	×	

It is also crucial to emphasize that in selecting the textbooks of our larger sample (see Appendix A), we considered publishers from various backgrounds, including Academic Press, Cambridge University Press, Edinburgh University Press, Harvard University Press, Hong Kong University Press, Mathematical Association of America Press, MIT Press, Oxford University Press/Clarendon Press, Princeton University Press, University of California Press, University of Notre Dame Press; Broadview Press, Chapman and Hall, CRC Press, Dover Publications, Hackett Publishing, North Holland/Elsevier, Pearson, Rowman & Littlefield, Routledge, Springer Science, Van Nostrand Reinhold Company, Walter de Gruyter, Waveland Press, Wiley, World Scientific Publishing Company; Éditions De Boeck Supérieur, Éditions de la Chenelière, Éditions Dunod, Éditions Economica, Éditions Ellipses, Éditions Gallimard, and Presses polytechniques et universitaires romandes.

By carefully selecting textbooks of varied natures published over a sufficiently long period, we aim, on the one hand, to ensure a balanced representation of multiple approaches and trends, especially in terms of communication or organization of content in target textbooks, and, on the other hand, to establish a solid analytical foundation for a nuanced and in-depth investigation.

Table 2 – Conjunctive dominance versus Disjunctive dominance

	Conjunctive dominance approach	Disjunctive dominance approach
25-textbook sample	52%	48%
199-textbook sample	53%	47%

A simple exploratory analysis of the information provided in Table 1 clearly reveals the lack of consensus in the literature on the order of stating De Morgan’s laws. Approximately 52% of authors state De Morgan’s laws by using the conjunctive dominance approach, while a significant percentage, nearly 48%, opt for the disjunctive dominance approach. This lack of consensus persists when expanding the sample to 199 textbooks (see Table 2 and Appendix A for further details). In this larger sample, it is observed that 53% of authors use conjunctive dominance, versus 47% who choose disjunctive dominance.

This lack of consensus becomes even more apparent as a more careful analysis of the data in Table 1 reveals cases where the same author indiscriminately applies either conjunctive dominance or disjunctive dominance from one publication to another. To illustrate this deep lack of consensus, we consider five isolated cases.

For instance, in his work published in 1972, Professor Herbert B. Enderton from UCLA states De Morgan’s laws using conjunctive dominance (see [Enderton 1972](#), p. 37). Five years later, he opts for disjunctive dominance (see [Enderton 1977](#), p. 31).

Another example concerns Professor Thomas Jech from Penn State. In a coauthored book, Thomas Jech and his coauthor Karel Hrbacek state De Morgan’s laws using conjunctive dominance (see [Hrbacek and Jech 1999](#), p. 14). However, in another book published eight years later, of which he is the sole author, he states De Morgan’s laws using disjunctive dominance (see [Jech 2007](#), p. 78).

The third example concerns Professor Patrick Suppes from Stanford University. In a book for which he is the sole author, De Morgan’s laws are stated following conjunctive dominance (see [Suppes 2012](#), p. 34). However, in a different book coauthored with Shirley Hill, published during the same year, De Morgan’s laws are stated following disjunctive dominance (see [Suppes and Hill 2012](#), pp. 98-99).

The last example concerns Professor Paul R. Halmos from the University of Chicago. In his work titled *Lectures on Boolean Algebras*, first published in 1963, and reissued in 1974 and 2018, Halmos states De Morgan’s laws using conjunctive dominance (see [Halmos 2018](#), p. 5). However, in another book titled *Naive Set Theory*, the author states De Morgan’s laws using disjunctive dominance (see [Halmos 1998](#), p. 20).⁶

The last example concerns the inconsistency that can arise even among books published by a same publishing house. For instance, in Table 1, when considering three textbooks published by Princeton University Press, it is observed that in one book, the author uses disjunctive dominance (see [Church 1996](#), p. 104), while the authors of the other two books use conjunctive dominance (see [Epstein 2011](#), p. 35, [Bernstein 2018](#), p. 22). Moreover, among the three books published by Cambridge University Press (see Table 1), it is noted that in two of them, authors use disjunctive dominance ([Hall 1966](#), p. 37, [Cunningham 2016](#), p. 10), while in the third book, the

6. Using data from Appendix A, we may provide many more examples. For instance, in Appendix A, we can observe another intriguing case in which the authors of the same book use both conjunctive dominance (see [Falkenstein et al. 2021](#), pp. 117-118, p. 122) and disjunctive dominance (see [Falkenstein et al. 2021](#), p. 221).

author uses conjunctive dominance (Hamilton 1988, p. 14).

Importantly, it is also noteworthy that this lack of consistency in stating De Morgan’s laws is not limited to a specific era. For instance, considering five books published in the 1990s, it is observed that in three of them, authors use disjunctive dominance (Church 1996, p. 104, Adamowicz and Zbierski 1997, pp. 16-17, Halmos 1998, p. 20), while in the third book, conjunctive dominance is applied (Hrbacek and Jech 1999, p. 14). This pattern remains consistent when examining a larger sample or a period other than the 1990s (see Table A-1 in Appendix A).

Furthermore, as illustrated in Table A-1 in Appendix A, there are many documents discussing De Morgan’s laws. This abundance poses a challenge for establishing a norm for a straightforward and appropriate order when stating these laws. Consequently, it is important to ensure the tractability of our results. To carry out our archaeological analysis successfully, the paper primarily relies on De Morgan’s book *Formal Logic: The Calculus of Inference, Necessary and Probable*, published in 1847. In this work, as noted by Beth (1947/1948, p. 271), De Morgan demonstrated a perfect knowledge of the logical tradition. This constitutes a credible archive from the perspective of scientific research.

Another important issue to address is explaining why it is important to define a norm in the statement of De Morgan’s laws. Providing a clear answer to this question illustrates the relevance of our study. Indeed, there are at least three fundamental reasons that justify the importance of specifying a norm in the statement of De Morgan’s laws.

First, from a scientific research perspective, the precision of the order in stating De Morgan’s laws has direct implications for how these laws are cited in the literature. Defining a clear reference to the first and second laws contributes to establishing a consistent benchmark in scientific communication, thereby improving harmony and collaboration within the scientific community.

Second, from an academic perspective, clarifying the order of De Morgan’s laws also has direct implications. By precisely establishing the first and second laws, one contributes to reinforcing rigor in the teaching of formal logic and, moreover, avoids potential confusion in the interpretation and application of these fundamental laws.

Finally, from a historical perspective, explicitly identifying the order of stating De Morgan’s laws offers a concise reference on the initial order in which these laws were formulated by their author. Such contextualization provides an authentic dimension to academic and scientific discourse.

3 Contextual and Historical Background

Most scientists, and students who have taken a course in logic, set theory, or discrete mathematics, are familiar with the name of Augustus De Morgan (1806-1871), a prominent figure among nineteenth century British mathematicians (Rice, 1999). De Morgan made significant contributions to logic and mathematics. His research in symbolic logic led to the emergence of relational logic, the introduction of term quantification, and the formulation of the famous De Morgan’s laws, which remain fundamental theorems in logic and set theory.⁷

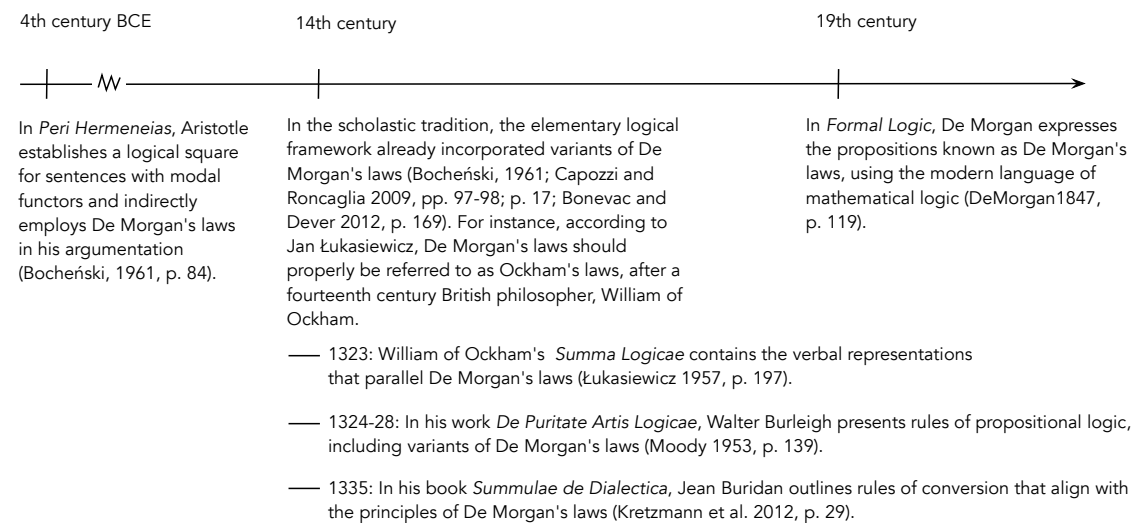
Figure 1 presents the evolution of De Morgan’s laws from the fourth century BCE to the nineteenth century. As far as we know, Aristotle was the first scholar to subtly incorporate the principles behind De Morgan’s laws within his logical square framework, providing an early glimpse into the logical interaction between conjunction, disjunction, and negation (Bocheński 1961, p. 84). Moreover, several manuscripts and discussions have revealed that in the fourteenth century, the medieval scholastic tradition had also integrated variants of De Morgan’s laws into its elementary logical framework (Bocheński 1961, p. 17; Capozzi and Roncaglia 2009, pp. 97-98; Bonevac and Dever 2012, p. 169). For instance, as early as 1323, William of Ockham’s *Summa Logicae* contained verbal representations that parallel De Morgan’s laws (Lukasiewicz 1957, p. 197). Thus, according to Jan Lukasiewicz, De Morgan’s laws should properly be called Ockham’s laws. After Ockham published *Summa Logicae*, Walter Burleigh wrote a tract on logic titled “De Puritate Artis Logicae” in which he presented rules of propositional logic, including variants of De Morgan’s laws (Moody 1953, p. 139). In the same vein, Jean Buridan, in his book titled *Summulae de Dialectica*, outlines rules of conversion that align with the principles of De Morgan’s laws (Kretzmann et al. 2012, p. 29). This chronological account offers an overview of how De Morgan’s laws were employed in logic before Augustus De Morgan formally expressed them in the nineteenth century. For further details, see Figure B-1 in Appendix B that sheds light the connection between De Morgan’s laws and reasoning in formal logic.

As the first mathematics professor at University College London, De Morgan is credited with coining the term *mathematical induction* (Murawski 2010, p. 247) and popularizing the four-color problem (Wilson 1999, p. 520). An exploration of any index on the history of logic and mathematics will undoubtedly reveal references to his contributions, highlighting his enduring impact on the mathematical field.

7. See, for instance, Whittaker (1944/1945), Bocheński (1961), Richards (1987), Merrill (1990), and the subsequent literature for further details.

Regarding logical symbolism, efforts to construct a formal logic system date back much earlier (see, for example, Descartes, Hobbes, Leibniz, Lambert, and so on. Indeed, one can trace it back to Aristotle). However, as argued by Beth (1947/1948, p. 274), the conceptualization of a program of symbolic logic (also referred to as *mathematical logic*, see Peckhaus 1999, p. 433) in its modern sense truly took shape around 1847. Notably, this occurred simultaneously with the publication of Augustus De Morgan's *Formal Logic* and George Boole's *The Mathematical Analysis of Logic*, in which both authors aimed to study logic from a mathematical perspective.⁸

Figure 1 – Timeline of De Morgan's laws: Exchange of conjunction and disjunction through negation



Whether he called it science or art,⁹ De Morgan (1847, ch. II, p. 26) defined logic as a branch of research that studies how the mind reasons, with a particular

8. As can be immediately observed, George Boole explicitly expresses such a stance in the titles of his works: "The Mathematical Analysis of Logic, Being an Essay towards a Calculus of Deductive Reasoning" and "An Investigation of the Laws of Thought on Which Are Founded the Mathematical Theories of Logic and Probabilities," published in 1847 and 1854, respectively. Additionally, as mentioned by Beth (1947, p. 331), the influence of Boole's and De Morgan's works on the subsequent development of formal logic can be fully understood through the biography established by Church (1936a,b,c,d, 1938, 1952). See also, Venn (1876b), Beth (1947/1948), and Heyting (1947/1948), among many others, for further details.

9. The use of the term "logic" dates back at least to Democritus, the author of a book titled *On The Logic or The Canons*. Logic, derived from the word *logike*, an adjective derived from *logos*, is

emphasis on the connection between thought and language.¹⁰

De Morgan's (1847) *Formal Logic* consists of 14 chapters (pp. 1-296) and two appendices. In the book (pp. 296-336), De Morgan addresses various aspects of logic, including propositions, syllogisms, truth, induction, and errors related to premises. The work delves into the evolution of logical jargon over time, providing insight into the distinctions between terms such as "fallacious," "sophism," "paradox," and "paralogism." Also, the author attempts to describe syllogism verbally, using positive terms for universal relations and their corresponding negative contraries. The first appendix presents the report of a controversy between De Morgan and Sir William Hamilton, professor of logic and metaphysics at the University of Edinburgh, along with the De Morgan's final reply to the latter (pp. 296-323). The second appendix addresses forms of inference that are different from those of the Aristotelians (pp. 323-336).

We are now ready to look in more detail at each chapter before seeking to carefully extract De Morgan's first and second laws.

In the first chapter of *Formal Logic*, De Morgan (1847, p. 1-25) considers logic as the examination of reasoning, ensuring that inference is true if the premises are, without concern for the truth of the facts, opinions, or presumptions from which the inference is drawn. Whether the premises are true or false is not a question of logic but of morality, philosophy, history, or any other field to which their object belongs. Consequently, logical inference or reasoning mainly involves comparing different things and determining points of resemblance or difference between them, which requires prior propositions. In this context, all propositions must be either affirmations or negations and can thus be reduced to simple forms such as " X is Y " or " X is not Y ."

In the second chapter, De Morgan (1847, p. 26-46) discusses basic issues of logic by emphasizing its origin. Derived from the Greek word " $\lambda\acute{o}\gamma\omicron\varsigma$," logic means the communication of thought, generally through speech. De Morgan thus addresses the complexity of studying the mind, comparing it to a mechanism like a watch,

often associated with *techné* (art) or *épistémè* (science). Since *logos* means discourse, argument, or reason, *logike techné* thus denotes the art of reasoning, and *logike episteme* refers to the science of reasoning. See Cellucci (2013, pp. 33-34) for more details.

10. In this specific context— see, for example, Keynes (1879, p. 364)— thought refers to the ability to apprehend relations, while language, according to Whately (1855, see Book I, p. 25), can be seen as an indispensable instrument for any reasoning that deserves its name, whether for refuting an opponent, conveying instructions, or satisfying one's own mind on any subject.

and insists that our knowledge of thought is limited to observable manifestations, without access to its internal structure. He also distinguishes certainties related to our individual existence from those related to the existence of other entities, thus introducing nuance into the understanding of reality.

In the third chapter, [De Morgan \(1847, p. 46-54\)](#) highlights the challenges related to the connection between the use of words and the notions of things, as well as the various theories and disputes arising from this interaction. The author also discusses the use of general symbols, such as alphabet letters, in logic and the need to consider them as symbols without worrying about what they represent.

In the fourth chapter, [De Morgan \(1847, p. 54-76\)](#) deals with the notion of propositions, describing them as affirmations of agreement or disagreement between two names. The author introduces the concept of *contraries* and specifies its analytical formalization. Indeed, if X is a name, then its contrary will be denoted by a lowercase letter, x .

In the fifth chapter, [De Morgan \(1847, p. 76-106\)](#) discusses the notion of syllogism. A syllogism is a strategy that involves drawing a conclusion about the relationship between X and Y based on each of these names' relationship with a third, Z . The author explores the limitations of classical syllogism by highlighting the absence of contrary terms and showing that this absence can lead to neglecting several legitimate modes of inference. More importantly, De Morgan introduces the idea of fundamental syllogisms, guaranteeing a certain parsimony in formulating premises.

In the sixth chapter, [De Morgan \(1847, p. 107-126\)](#) addresses the truth and falsehood of syllogisms, emphasizing that when the premises of a syllogism are true, the conclusion is also true, and when the conclusion is false, at least one of the premises is false. The author also discusses necessary and sufficient conditions, noting that necessary conditions are those without which the thing cannot be, while sufficient conditions are those with which the thing must be.

In the seventh chapter, [De Morgan \(1847, p. 127-141\)](#) addresses the principles of the Aristotelian syllogistic and the exclusions applied to this method of inference.¹¹ The author emphasizes two principles of exclusion, one related to the *dictum de omni et nullo* (i.e., everything predicated of a whole is predicated of every part of that whole, whether it is an affirmation or a negation) and the other concerning the exclusion of contrary terms. De Morgan questions these exclusions, asserting that the

11. See [Dopp \(1952\)](#), [Duerlinger \(1968\)](#), and [Barreau \(2012\)](#) for a modern exposition of the Aristotelian syllogistic.

first is an error of substitution, confusing the determination of a natural limit with the assertion that no other limit can exist. As for the exclusion of contraries, De Morgan thinks it probably stems from a gap in understanding the scope of argumentation. De Morgan thus proposes a more flexible approach, arguing that the simple idea of negation should not necessarily be attached to the contrary.

In the eighth chapter, [De Morgan \(1847, p. 141-170\)](#) explores numerically defined syllogisms and introduces the idea of numerical definition of universal propositions. He emphasizes the crucial distinction between the distributive truth and collective truth of propositions, insisting that the proposition “every X is Y ” is distributively true when every X is individually considered as Y .¹² Moreover, the proposition “every X is Y ” is collectively true when all X form Y ; that is, when one does not mean that any X is Y or that any number of X are Y , but rather, one wants to express that all X together form Y .

In the ninth chapter, [De Morgan \(1847, p. 170-191\)](#) emphasizes the difficulty of determining the essence of things and claims that we do not know what we are talking about when we pose such a question. Thus, the author proposes a graduation of knowledge into high knowledge and necessary truths: (i) *High knowledge*. What we know, of which we are certain and convinced that nothing could persuade us otherwise, is the existence of our own minds, thoughts, and perceptions. This high knowledge, this absolute certainty, admits no imagination regarding the possibility of falsehood; and (ii) *Necessary truths*. Alongside high knowledge come things of which we can only think if we are as certain of them as of our own existence. These are called necessary truths. While the absolute and inexhaustible feeling of knowledge denotes *certainty*, lower levels of knowledge generally refer to degrees of belief or simply degrees of knowledge. Thus, in De Morgan’s sense, probability implies belief, and belief is just another name for imperfect knowledge.

In the tenth chapter, [De Morgan \(1847, p. 191-210\)](#) examines the sources of conviction; namely, argumentation and testimony, qualifying them as statements by demonstration and statements by authority, respectively. Indeed, according to De Morgan, since an argument is a consequence of proof, its invalidity is only that of the demonstration, even if the conclusion may still be true.¹³ Furthermore, testimony being an assertion of the truth of the conclusion, its invalidity can only be that of the truth of the conclusion. Building on this distinction, De Morgan explores

12. Distributive means universally (see [De Morgan 1847, p. 139](#)).

13. For example, if a proposition by Euclid is poorly demonstrated, it does not necessarily mean that the statement is false.

how to measure the strength of testimony compared to an argument and develops a probability-based approach to evaluate the validity of sources of conviction.

In the eleventh chapter, [De Morgan \(1847, p. 211-226\)](#) addresses the question of induction, a method of reasoning that involves drawing a general conclusion from specific observations, called “specimens” or “instances.” The author explains that inductive demonstration has a strictly syllogistic nature; that is, it follows a logical form of reasoning. In addition, De Morgan introduces the concept of induction by connection, which is more common in mathematics, and discusses inductions based on probability. He emphasizes that in certain circumstances, although formal demonstration is difficult to achieve, inductions based on probability can offer a high moral certainty.

In the twelfth chapter, [De Morgan \(1847, p. 227-237\)](#) explores the evolution of jargon used in logic, noting, for example, that the distinction between logic and dialectic remained unclear and that terms such as “discourse” and “argument” evolved. He also addresses Aristotle’s logical categories, such as substance, quantity, quality, relation, action, and passion. De Morgan explains how these metaphysical notions were considered as fundamental in logic teaching at the time.

In the thirteenth chapter, [De Morgan \(1847, p. 237-286\)](#) addresses errors related to premises and questions where it is not evident whether the error lies in the matter or in the form of reasoning. He emphasizes that detecting fallacies of inference is possible by respecting the rules, but errors related to the matter of premises require particular attention. The author also mentions the varied use of terms like “fallacious,” “sophism,” “paradox,” and “paralogism,” noting that these can be applied to either logical errors or factual errors, and he discusses different conceptions of these terms. Finally, he specifies that the question of the validity of premises in terms of fact or principle does not fall within the realm of logic but rather within specific domains of knowledge such as botany, astronomy, and theology.

In the last chapter, [De Morgan \(1847, p. 286-296\)](#) attempts to describe the syllogism using positive terms for universal relations and their corresponding negative contraries. He introduces terms such as “genus” and “species” to describe relations between syllogism terms, proposing substitutions to express relations in different contexts.

4 Statements of De Morgan's Laws

In this section, we intend to carefully state De Morgan's laws, taking into account the terminology and notations used by the author in *Formal Logic*. To make our exposition more tractable, we will provide references and page numbers for each key passage.

To do this, let's begin by clarifying what is meant by a *name*. By definition (see [De Morgan 1847](#), p. 35), a *name* refers to an idea that can be affirmative or negative. In the field of logic, authors have successfully replaced specific names with simple letters of the alphabet, such as A, B, C , or P, Q, R , or even X, Y, Z , and so on (see [De Morgan 1847](#), p. 47). For example, "every man is an animal" can be expressed as "every P is Q ." This convention simplifies the expression of logical relationships by eliminating specific names, allowing for a more abstract analysis of inference laws. Therefore, a name can be seen as a symbol that is associated with one or more objects of thought due to a resemblance or a set of properties. When one assigns agreement or disagreement between two names, a proposition is obtained ([De Morgan 1847](#), p. 54). More explicitly, following [Venn \(1876a, p. 44\)](#), a proposition can be defined as the statement of a judgment about things.¹⁴ Since contraries, by construction, are designated by uppercase and lowercase letters,¹⁵ it follows that a proposition can only be either X or x but never both at the same time.

Consider three specific names: P, Q , and R . It is possible to form complex terms either conjunctively or disjunctively and characterize the negation for each of them. Following [De Morgan \(1847, pp. 115-116\)](#), if our goal is, for example:

1. To give a common name to all three instances, then we can denote it as " PQR ";
2. To give a name to anything that is either of the three instances (i.e., one or more of them), then we can write " P, Q, R ";

14. In this definition, three keywords draw attention. First, the words in which the proposition is expressed. Second, the judgment it expresses. Third, the reference to the things contained therein. Thus, in his article, John Venn specifies that: (i) without the *words*, there could be nothing more than the simplest germ of thought; (ii) without the *judgment* it expresses, a proposition would obviously not be an action emanating from a rational being; (iii) without the reference to *things*, a proposition would, of course, fail in its main objective of communicating knowledge, and no question could be raised regarding its truth or falsehood. See [Whately \(1855, Book II\)](#) and [Keynes \(1879\)](#), among others, for further details.

15. That is, for a name X , the lowercase letter x denotes its contrary name (see [De Morgan 1847](#), p. 56).

3. To give a name to instances P and Q together or to R , then we can write “ PQ, R .”

Indeed, while the expression “ PQR ” denotes the *conjunction* and thus includes instances common to P , Q , and R , the expression “ P, Q, R ” denotes the *disjunction* and includes either instance P , instance Q , instance R , or instances common to: P and Q ; P and R ; Q and R ; P , Q , and R respectively. It is clear that the disjunctive term “ P, Q, R ” will always be strictly broader than the conjunctive term “ PQR ,” unless instances P , Q , and R are all identical.

Building on the above, [De Morgan \(1847, p. 117\)](#) introduces the following statements, which closely resemble the first historical formulation of De Morgan’s laws:

1. The negation of “ PQR ” is “ p, q, r ”;
2. The negation of “ P, Q, R ” is “ pqr ”;
3. The negation of “ PQ, R ” is “ $(p, q)r$.”

This means the negation of conjunction is disjunction and the negation of disjunction is conjunction. Furthermore, [De Morgan \(1847, p. 118\)](#) later demonstrates that these three statements can be reduced to two assertions. Let “ PQ ” be a complex term that connects by disjunction the names P and Q (i.e., includes both names). Let “ P, Q ” be a complex term that connects by conjunction the names P and Q (i.e., includes at least one of the two names). Then:

De Morgan’s first law : The negation of “ PQ ” is “ p, q ”;

De Morgan’s second law : The negation of “ P, Q ” is “ pq .”

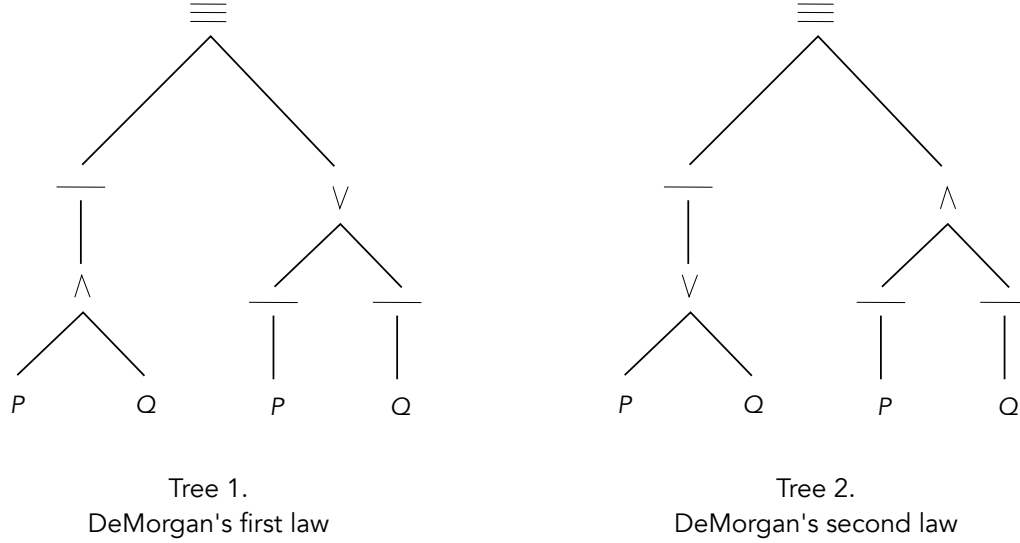
Using modern notations, it follows that:

$$\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q} \quad (\text{De Morgan’s first law}) \text{ and}$$

$$\overline{P \vee Q} \equiv \overline{P} \wedge \overline{Q} \quad (\text{De Morgan’s second law}),$$

respectively, with: $\overline{P} \equiv p$; $\overline{Q} \equiv q$; $P \wedge Q \equiv PQ$; $P \vee Q \equiv P, Q$; $\overline{P} \wedge \overline{Q} \equiv pq$; $\overline{P \vee Q} \equiv p, q$. Thus, while De Morgan’s first law implies that the negation of a conjunctive complex term is equivalent to the disjunction of the negations of their atomic names, De Morgan’s second law states that the negation of a disjunctive complex term is equivalent to the conjunction of the negations of their atomic names.

Figure 2 – Logic trees of De Morgan’s first and second laws



Upon revisiting the literature in light of our findings, it therefore appears that the order in stating De Morgan’s laws, as considered, notably, by [Leung and Chen \(1967, p. 36\)](#), [Stolyar \(1971, p. 60\)](#), [Quine \(1982, p. 57\)](#), [Epstein \(2011, p. 35\)](#), and more recently [Bernstein \(2018, p. 22\)](#) and [Smith \(2022, p. 139\)](#), aligns with the historical evidence we have provided.

Meanwhile, the order in stating De Morgan’s laws considered by authors such as [Bocheński \(1960, pp. 20-21\)](#), [Stoll \(1963, p. 20\)](#), [Enderton \(1977, p. 31\)](#), [Adamowicz and Zbierski \(1997, pp. 16-17\)](#), [Cori and Lascar \(2000, p. 31\)](#), [Chiswell and Hodges \(2007, p. 71\)](#), [Corbae et al. \(2009, p. 19\)](#), [Cunningham \(2016, p. 10\)](#), [Allen and Hand \(2022, pp. 28-29\)](#), [Salim \(2023, p. 20\)](#), and many others, including *The Concise Oxford Dictionary of Mathematics* (see [Clapham and Nicholson 2009, p. 118](#)), is not consistent with the historical evidence that we have documented.

In first-order logic, the fundamental objects of study are terms, formulas, and proofs or derivations. While terms denote the objects whose properties we want to prove, formulas represent the properties of the objects under study. Proofs allow for establishing whether a formula is true or not. In a formula, terms can be connected by symbols or operators.¹⁶ A formula connected by symbols is said to be complex.

16. There are two main classes of symbols—On the one hand, logical connectives (mainly including

Otherwise, it is considered an atomic formula. Importantly (see, e.g., [Bourbaki 2006](#), p. E I.10), symbols not only allow the construction of complex formulas but also characterize a language.¹⁷

Since a formula can be atomic or complex, it is sometimes more intuitive to represent a complex formula as a tree where each node is labeled by a symbol and each leaf by an atomic formula or a formula of lower-order complexity. Thus, as illustrated in Figure 2, De Morgan’s first and second laws can naturally be expressed in the form of logic trees.

5 Conclusion

In this paper, we have documented the lack of a clear consensus on the statement order of De Morgan’s laws in logic and mathematics textbooks. We have highlighted the potentially detrimental impact of this disparity on harmony in scientific communication and academic discussions. This observation underscores the importance of identifying a plausible criterion that can facilitate the emergence of consensus in the future. To achieve this, we have implemented a simple strategy that involves retracing the first modern formulation of De Morgan’s laws and extracting, from this archaeological exercise, a straightforward criterion that establishes the original order in stating these laws as the norm to be rigorously considered henceforth in textbooks, scientific communication, and academic discussions. We hope that this simple yet sufficiently plausible criterion can contribute, to some extent, to establishing a norm within the academic and scientific communities and, furthermore, encourage more coherent communication regarding these fundamental laws.

unary and binary operators) and, on the other hand, mathematical quantifiers (mainly including universal and existential quantifiers).

17. By *language*, also called vocabulary or signature, we mean the specification of a collection or family of symbols. See [Kreisel and Krivine \(1967, p. 16\)](#), [Gamut \(1991, pp.1-5\)](#), [David et al. \(2001, p. 11\)](#), and [Crossley et al. \(2012, p. 1\)](#), among others, for further details.

Appendix

A Assessing the Statement Order of De Morgan's Laws in a Large Sample

For enhanced clarity, Table A-1 is organized chronologically. In cases where textbooks share the same publication year, we have carefully classified them in alphabetical order. Finally, for each bibliographic entry, we have provided the specific page in which the author or authors explicitly state De Morgan's laws.

Table A-1 – Order of stating De Morgan's laws in a sample of 199 logic and mathematics textbooks

Reference and target page	Documentation 1	Documentation 2
	St. 1: $\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q}$	St. 1: $\overline{P \vee Q} \equiv \overline{P} \wedge \overline{Q}$
	St. 2: $\overline{P \vee Q} \equiv \overline{P} \wedge \overline{Q}$ <i>Conjunctive dominance</i>	St. 2: $\overline{P \wedge Q} \equiv \overline{P} \vee \overline{Q}$ <i>Disjunctive dominance</i>
Bocheński (1960, pp. 20-21)		×
Stoll (1963, p. 20)		×
Hall (1966, p. 37)		×
Leung and Chen (1967, p. 36)	×	
Kreisel and Krivine (1967, p. 10, 17)		×
Robbin (1969, p. 13)	×	
Bick (1971, p. 8)	×	
Stolyar (1971, p. 60)	×	
Enderton (1972, p. 37)	×	
Rasiowa (1973, p. 217)	×	
Barwise (1977, p. 25)	×	
Curry (1977, p. 293)	×	
Enderton (1977, p. 31)		×
Loeve (1977, p. 57)		×
Johnstone (1979, p. 479)	×	
Kilgore (1979, p. 227)		×
Barwise (1982, p. 25)	×	

Kramer (1982, p. 110)		×
Murphy (1982, p. 19)	×	
Quine (1982, p. 57)	×	
Cooke and Bez (1984, p. 174)	×	
Molluzzo and Buckle (1986, p. 95, 353)		×
Holdsworth (1987, p. 12)	×	
Hamilton (1988, p. 14)	×	
Gustason and Ulrich (1989, pp. 19-20)	×	
Joshi (1989, p. 59)		×
Landin (1989, p. 13)		×
Chang and Keisler (1990, p. 38)		×
Lucas (1990, p. 19)		×
Fletcher et al. (1991, p. 234)	×	
Gamut (1991, p. 48)		×
Piff (1991, p. 54)		×
Kolmogorov and Yushkevich (1992, p. 31)	×	
Munro (1992, p. 74)	×	
Cohen and Nagel (1993, p. 70)		×
Giri (1993, p. 40)		×
Goodman (1993, p. 70)		×
Ebbinghaus et al. (1994, p. 35)		×
Foldes (1994, p. 22)	×	
MacLane and Moerdijk (1994, p. 50)		×
Moschovakis (1994, p. 5)		×
Whitelaw (1995, p. 17)	×	
Avriel and Golany (1996, p. 139)	×	
Balakrishnan (1996, p. 6)	×	
Church (1996, p. 104)	×	
Troelstra and Schwichtenberg (1996, p. 72)	×	
Adamowicz and Zbierski (1997, pp. 16-17)		×
Bostock (1997, p. 46)	×	
Howson (1997, p. 40)		×
Mendelson (1997, p. 23)		×

Ash (1998, p. 4)	×	
Cauman (1998, pp. 80-81)		×
Crowe and Hayes-Gill (1998, p. 7)		×
Halmos (1998, p. 20)		×
Halmos and Givant (1998, pp. 32-33)	×	
Simpson (1998, p. 229)		×
Bacon et al. (1999, p. 31)	×	
Cameron (1999, p. 70)		×
Hrbacek and Jech (1999, p. 14)	×	
Michel (1999, p. 6)	×	
Penner (1999, p. 54)	×	
Restall (1999, p. 63)		×
Rong Li (1999, p. 14)		×
Tomassi (1999, p. 161)	×	
Ablamowicz and Fauser (2000, p. 234)	×	
Cantrell (2000, p. 71)		×
Cori and Lascar (2000, p. 31)		×
Hummel (2000, p. 19)	×	
Vaught (2001, p. 13)		×
Andrews (2002, p. 59)	×	
Beichelt and Fatti (2002, p. 2)		×
Kleene (2002, p. 16)		×
Krantz (2002, p. 51)		×
Rosen (2002, p. 15)	×	
Ayyub (2003, p. 457)		×
Devlin (2003, p. 63)		×
Forster (2003, p. 74)	×	
Goldfarb (2003, p. 18)	×	
Robinson (2003, p. 3)		×
Tourlakis (2003, p. 249)	×	
Wallis (2003, p. 36)		×
Koshy (2004, p. 24)	×	
Menini and Oystaeyen (2004, p. 12)		×

Soong (2004, p. 11)		×
Burgess (2005, p. 741)	×	
Gatti (2005, p. 310)		×
Hinman (2005, p. 34)	×	
Nolt et al. (2005, p. 54)	×	
Beichelt (2006, p. 2)		×
Gubner (2006, p. 12)	×	
Jeffrey (2006, p. 143)		×
Rautenberg (2006, p. 9)	×	
Ross (2006, p. 31)		×
Velleman (2006, p. 21)	×	
Chiswell and Hodges (2007, p. 71)		×
Jech (2007, p. 78)		×
Ok (2007, p. 6)		×
Priest et al. (2007, pp. 41-43)	×	
Ehrenfeucht et al. (2008, p. 55)	×	
Gowers et al. (2008, p. 188)	×	
Priest (2008, p. 145)	×	
Rosenkrantz (2008, p. 76)		×
Corbae et al. (2009, p. 19)		×
Garnier and Taylor (2009, p. 23)		×
Givant and Halmos (2009, p. 9, 45)	×	
Johnston and McAllister (2009, p. 20)	×	
Roberts (2009, p. 16)	×	
Bird (2010, pp. 544)		×
Eie and Chang (2010, p. 6)		×
Wheeler and Brawner (2010, p. 16)		×
Whitesitt (2010, p. 8)	×	
Arnold (2011, p. 73)		×
Ayyub and McCuen (2011, p. 70)		×
Barker-Plummer et al. (2011, p. 83)	×	
Bloch (2011, p. 20)	×	
Epstein (2011, p. 35)	×	

Goldblatt (2011, p. 205)		×
Ben-Ari (2012, p. 26)		×
Crain (2012, p. 53)		×
Cunningham (2012, p. 8)		×
DeLong (2012, p. 105)		×
Gensler (2012, p. 120)	×	
Jacod and Protter (2012, p. 9)		×
Khisty et al. (2012, p. 131)		×
Kochendörffer (2012, p. 15)	×	
Krantz (2012, p. 7)		×
Lavrov and Maksimova (2012, p. 5)	×	
Makinson (2012, p. 20)		×
Nerode and Shore (2012, p. 36)		×
Parkes (2012, p. 284)	×	
Shynk (2012, p. 55)		×
Smith (2012, pp. 131-132)	×	
Suppes (2012, p. 34)	×	
Suppes and Hill (2012, pp. 98-99)		×
Walicki (2012, p. 21)	×	
Agler (2013, p. 213)		×
Dalen (2013, p. 20)		×
Dasgupta (2013, p. 14)		×
Gabbay and Wansing (2013, p. 14)	×	
Gavrilov and Sapozhenko (2013, p. 11)	×	
Hodel (2013, p. 73)	×	
Hogg et al. (2013, p. 6)	×	
Jenkyns and Stephenson (2013, p. 84)	×	
Pudlák (2013, p. 21)	×	
Tarski (2013, p. 52)	×	
Yaqub (2013, p. 158)	×	
Caulier (2014, p. 14)	×	
Copi et al. (2014, p. 349)		×
Nagla (2014, p. 122)		×

Taylor and Garnier (2014, p. 39)	×	
Conradie and Goranko (2015, p. 121)	×	
Gregory (2015, p. 18)		×
Hayek and Leca (2015, p. 7)		×
Liesen and Mehrmann (2015, p. 21)		×
O’Leary (2015, p. 35)	×	
Arthur (2016, p. 103)		×
Bodin et al. (2016, p. 18)	×	
Cunningham (2016, p. 10)		×
Goranko (2016, p. 28)	×	
Harris and Harris (2016, p. 63)	×	
Hurley (2016, p. 317)	×	
Ferland (2017, p. 13)	×	
Gregory (2017, p. 136)	×	
Bernstein (2018, p. 22)	×	
Byer et al. (2018, p. 68)	×	
Don (2018, p. 44)	×	
Halmos (2018, p. 5)	×	
Kreyszig (2018, p. 1018)		×
Kurgalin and Borzunov (2018, p. 6)	×	
Bonheur (2019, p. 68)	×	
Farlow (2019, p. 12)		×
Lewis and Zax (2019, p. 108)	×	
Ponomarenko (2019, p. 26)	×	
Singh (2019, p. 835)		×
Haghighi and Wickramasinghe (2020, p. 8)		×
Patarin (2020, p. 153)	×	
Smith (2020, p. 113)	×	
Easttom (2021, p. 99)	×	
Erciyes (2021, p. 12)		×
Falkenstein et al. (2021, pp. 117-118, p. 122)	×	
Falkenstein et al. (2021, p. 221)		×
Hu (2021, p. 11-12)	×	

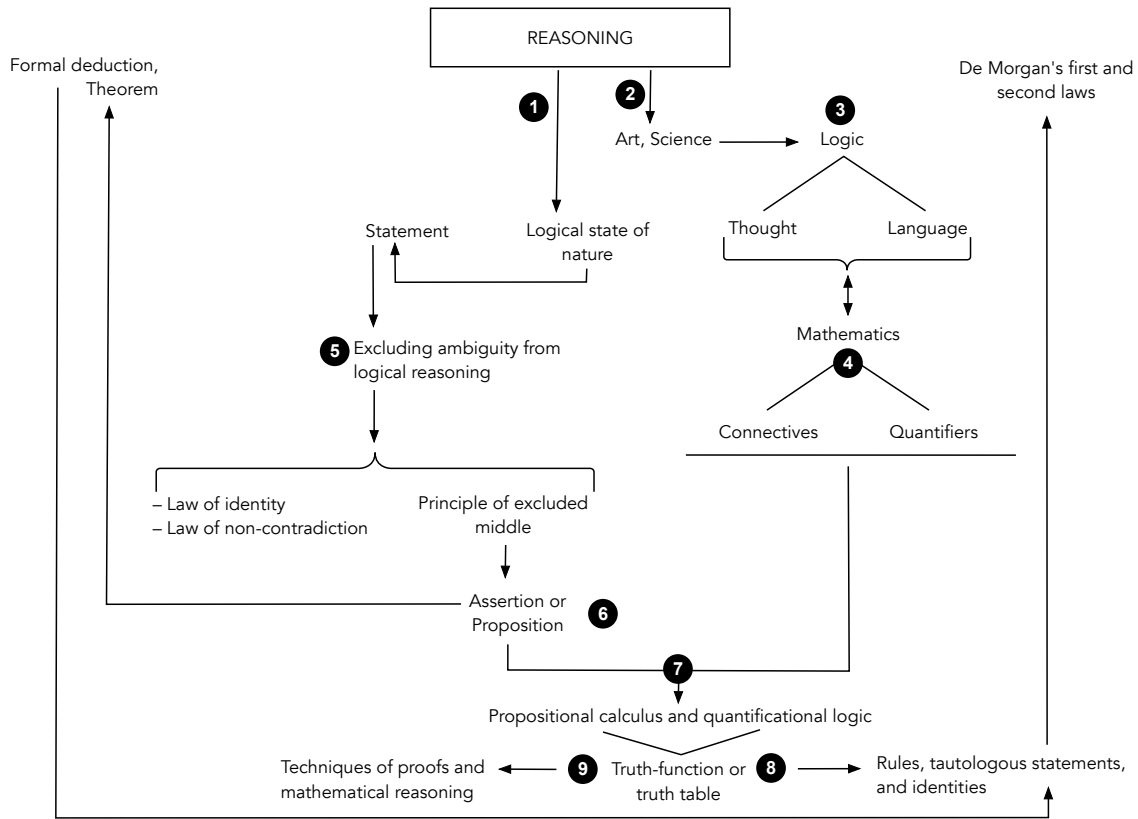
Jebril et al. (2021, p. 46)		×
Parkers (2021, p. 7)	×	
White and Ray (2021, p. 29)	×	
Allen and Hand (2022, pp. 28-29)		×
Avigad (2022, p. 48)	×	
Beziau and Vandoulakis (2022, p. 142)		×
Cormen et al. (2022, p. 1155)	×	
Le Barbenchon et al. (2022, p. 37)	×	
Polkowski (2022, p. 3)	×	
Smith (2022, p. 139)	×	
de With (2023, p. 629)		×
Hédi (2023, p. 10)		×
Hu (2023, p. 21)	×	
Milanič et al. (2023, p. 36)		×
Salim (2023, p. 20)		×
Sarkar and Prasad (2023, p. 39)	×	
Yadav (2023, p. 40)		×

B De Morgan Laws in a Basic Formal Logic Framework

The study of the relationship between logic, mathematics, and intellectual development has a rich historical background (for further discussions, see [Joseph 1928](#), [Quine 1937](#), [DeStefano 1973](#), [Jansson 1986](#), [McGinn 1993](#), [Brakes 2001](#), [Sandu and Hyttinen 2001](#), and [Wagner 2018](#) among others). This section specifically sheds light the inherent connection between De Morgan’s laws and logical reasoning.

Man is commonly depicted as an animal endowed with superior rationality compared to other species ([Ritter 1930](#), p. 472). Reasoning, defined as a process of revising beliefs, altering perspectives, and adjusting intentions and plans ([Harman 1984](#), 107; [Goldstein 1988](#), p. 298), plays a crucial role in this rationality. The field of logic, also known as the art or science of reasoning, provides an in-depth perspective on this human faculty. According to [Cellucci \(2013, pp. 33-34\)](#), logic can be viewed as either the art of reasoning (*logike techne*) or as the science of reasoning (*logike techne*). Moreover, [De Morgan \(1847, chap. II, p. 26\)](#) emphasizes the crucial role of

Figure B-1 – Connection between De Morgan's laws and logical reasoning



logic in understanding the relationship between *thought* and *language*.

While language serves as an indispensable instrument for any form of reasoning (Whately 1855, cf. book I, p. 25), thought, defined as the ability to apprehend relations (see Keynes 1879, p. 364), requires language to be expressed. In this perspective, mathematics emerges as a prominent language due to its axiomatic foundations which exclude ambiguity. Mathematics indeed relies on the three traditional laws of thought: the law of identity, the law of non-contradiction, and the principle of excluded (see, for instance, Russell 1912, chap. 7; Swabey 1923; Hooper 1923; Kornblith 1992; Glymour 2015).

A statement conforming to the three classical laws of thought, especially the prin-

ciple of excluded, ¹⁸ is commonly referred to as an assertion or proposition. Moreover, the label “theorem” is assigned to a proposition that consistently holds true (Bourbaki 2006, p. E I.22 and Quine 1982, p. 319). Furthermore, propositions can be classified as atomic or complex. A complex proposition can be formed by combining atomic propositions through the use of symbols (i.e., connectives or quantifiers). An atomic proposition is an indivisible statement that does not contain any logical connectives or symbols. The truth-function or truth table thus emerges as a crucial tool to illustrate the construction of complex propositions and demonstrate various approaches of mathematical reasoning (Post 1921, Wittgenstein 1921).

In contrast to what we term the “logical state of nature,” truth is essential in logical reasoning, giving rise to the concept of truth-values or truth-possibilities in Wittgenstein’s (1921) terminology. Finally, as depicted in Figure B-1, the truth-function plays a crucial role in establishing truths, such as De Morgan’s first and second laws.

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18. Existence proofs carried out with the help of the principle of excluded middle usually are especially attractive because of their surprising brevity and elegance. Taking the principle of excluded middle from the mathematician would be the same, proscribing the telescope to the astronomer or to the boxer the use of his fists. To prohibit existence statements and the principle of excluded middle is tantamount to relinquishing the science of mathematics altogether. For, compared with the immense expanse of modern mathematics, what would the wretched remnants mean, the isolated results, incomplete and unrelated, that the intuitionists naïve obtained without the use of the logical ε -axiom? (Hilbert 1928, p. 16).

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